Estimation of co-seismic mass change in static gravity field recovery

Author: Andreas Kvas, kvas@tugraz.at

This document gives a brief overview of how co-seismic mass variations were co-estimated in ITSG-Grace2018s. The main idea is that a step function is estimated in regions where co-seismic mass change is expected, thus improving the description of the temporal changes in Earth's gravity field. The methodology is exemplified on the basis of a single earthquake dividing the whole observation time span into two intervals $i = \{1, 2\}$, but can be generalized to any number of intervals in a straightforward manner. For each interval we assemble the observation equations

$$\mathbf{l}_i = \mathbf{A}_i \mathbf{x}_i + \mathbf{e}_i \qquad \mathbf{e}_i \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_i) \tag{1}$$

with $\mathbf{l}_i$ beeing the observation vector, $\mathbf{A}_i$ the design matrix, $\mathbf{x}_i$ the static gravity field parameters, and $\mathbf{e}_i$ the residual vector. We then form the blocked system of observation equations for the whole observation time span

$$\begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \\ & \mathbf{A}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix}. \tag{2}$$

The next step is to perform the parameter transformation

$$\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{I} \\ & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{z} \\ \mathbf{x} \end{bmatrix}, \tag{3}$$

where $\mathbf{I}$ is a identity matrix of appropriate dimension, $\mathbf{x}$ is the static gravity field for the whole time span, and $\mathbf{z}$ is the correction for interval $i = 1$. Substituting (3) into (2) yields

$$\begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_1 \\ & \mathbf{A}_2 \end{bmatrix} \begin{bmatrix} \mathbf{z} \\ \mathbf{x} \end{bmatrix} + \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix}. \tag{4}$$

Since both $\mathbf{x}$ and $\mathbf{z}$ are global representations of Earth's gravity field, the functional model (4) would result in a loss of redundancy in regions where no co-seismic change occured. To counteract this over-parametrization, we introduce the pseudo-observations

$$\mathbf{0} = \mathbf{W}\mathbf{z} + \mathbf{w} \qquad \mathbf{w} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_{\mathbf{w}}), \tag{5}$$

where $\mathbf{W}$ is a window matrix covering Earth's surface expect for the region where a co-seismic change is expected. The combined normal equation system has the structure

$$\begin{bmatrix} \mathbf{N}_1 + \mathbf{W}^T \mathbf{\Sigma}_{\mathbf{w}}^{-1} \mathbf{W} & \mathbf{N}_1 \\ \mathbf{N}_1 & \mathbf{N}_1 + \mathbf{N}_2 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{z}} \\ \hat{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_1 + \mathbf{n}_2 \end{bmatrix}. \tag{6}$$

Increasing the weight of the constraint by $\mathbf{\Sigma}_{\mathbf{w}} \rightarrow 0$, which in practice is done by scaling with a number close to numerically zero, then allows signal in $\hat{\mathbf{z}}$ only in regions within the predfeined area. This retains the redundancy in points which are not affected by the earthquake, thus not influencing the estimate $\hat{\mathbf{x}}$.