The ocean tide model EOT11a in spherical harmonics representation

Technical Note

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1 The model EOT11a

Thirteen tidal constituents have been derived from altimetry data [1]. To complement the tidal spectrum, six long periodic waves are additionally included, which are either taken from FES2004 [2], or computed from the HW95 Tide Generating Potential catalogue [3] in a gravitational self-consistent way according to the sea-level equation. In this spherical harmonics representation, the S1 tide is excluded since it is mainly a radiational tide and its influence on ocean mass redistribution is not clarified yet. The eighteen provided tidal waves are summarized in the following table.

Table 1: List of main waves in EOT11a spherical harmonics representation

Darwin notation	**Doodson number**	**Source**
Long period waves		
Ω_1	055.565	HW95
Ω_2	055.575	HW95
S_a	056.554	HW95
S_{sa}	057.555	HW95
M_m	065.455	EOT11a
M_f	075.555	EOT11a
M_{tm}	086.455	FES2004
M_{sqm}	093.555	FES2004
Diurnal waves		
Q_1	135.655	EOT11a
O_1	145.555	EOT11a
P_1	163.655	EOT11a
K_1	165.555	EOT11a
Semi-diurnal waves		
$2N_2$	235.755	EOT11a
N_2	245.655	EOT11a
M_2	255.555	EOT11a
S_2	273.555	EOT11a
K_2	275.555	EOT11a
Quarter-diurnal waves		
M_4	455.555	EOT11a

2 Matlab tide prediction software

For the convenience of the users, Matlab routines are supplied to perform tidal analysis with EOT11a. The formalism for tidal analysis and prediction is summarized in Appendix A.

Routine *main.m*:

The computations are performed in the routine main.m, where following relevant settings at the beginning of the file can be changed:

Table 2: Settings in the provided *main.m* routine

maxDegree	Maximum degree and order of the series expansion to be evaluated
mjd	Time of evaluation given as Modified Julian Date

To assure a fast computation, for each tidal wave two sets of corresponding potential coefficients *cnmCos*, *snmCos* and *cnmSin*, *snmSin* (including the loading potential) up to maximum degree and order 120 are provided in the ICGEM format for Earth Gravity Field Models [4] in the subfolder *eot11a/*. These files are read by the subfunction *readPotentialCoefficients.m*. The stokes coefficients for the use in equation (7) of Appendix A are obtained by

$$\Delta c_{nm,s} = cnmCos\cos(\Theta_s) + cnmSin\sin(\Theta_s)$$
$$\Delta s_{nm,s} = snmCos\cos(\Theta_s) + snmSin\sin(\Theta_s)$$

The astronomical argument Θ_s for each tidal wave is computed in subfunction *doodsonArguments.m*. Note that the Doodson-Warburg phase corrections χ_s (cf. Appendix A) are already applied to the provided coefficients and therefore do not have to be considered.

For the evaluation of the tidal potential according to equation (7), the necessary associated Legendre functions are retrieved by the subfunction *legendreFunctions.m*. In case of evaluating tidal heights as performed within these routines in subfunction *plotWaterHeight.m*, the norming factor $\frac{GM}{R}$ in equation (7) changes to $\frac{M}{4\pi R^2 \rho_w} \frac{2n+1}{1+k'_n}$. The corresponding load Love numbers [5] are read from the *data/* subfolder.

Routine *mainWithAdmittance.m*:

The previously described routine *main.m* computes the tidal potential or water heights only of the main tidal waves given in EOT11a. However, there exist significant influences of additional minor tide constituents that are not included in the tide model, which should not be neglected in satellite geodesy especially for Low Earth Orbiters. Here we follow the theory of admittance and assume that the admittance (i.e. the relation of the tidal height with respect to the amplitude of the corresponding Tide Generating Potential TGP for a specific tidal wave) is a smooth function of the frequency, which is only slowly varying within a frequency band (cf. [6]).

Based on this assumption, minor tide constituents are linearly interpolated for long periodic (from Ssa, Mm, Mf, Mtm, Msqm), diurnal (from Q1, O1, K1) and semi-diurnal (from 2N2, N2, M2, K2) frequency bands separately according to [7] or [8]. For the choice of the minor tides to be considered, we introduced an empirical threshold of $2\text{x}10^{-4}$ m^2/s^2 of TGP amplitude, which leads to 238 additionally included minor tides.

The interpolation coefficients are provided in a simple matrix in the *data/* subfolder. In this way, the routine *mainWithAdmittance.m* evaluates the tidal height of altogether 256 tides (18 major tides from EOT11a and 238 interpolated minor tides).

3 Alternative representation

In addition to the provided internal format, the EOT11a is also published according to the IERS conventions 2010 [8]. Hence, the coefficients, amplitudes and phases correspond to $C^{\pm}_{nm,s}$, $S^{\pm}_{nm,s}$ and $\hat{C}^{\pm}_{nm,s}$, $\epsilon^{\pm}_{nm,s}$ of equation (6). Note that coefficients and amplitudes are dimensioned in [cm], while phases are given in [°]. Therefore, the evaluation of either equation (4) or (5) directly leads to tidal heights.

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References

[1] Savcenko R. and Bosch W. EOT11a – a new tide model from Multi-Mission Altimetry. Poster, October 2011. OSTST Meeting San Diego 19-21 October 2011.

[2] Lyard F., Lefevre F., and Letellier T. Modelling the global ocean tides: modern insights from FES2004. *Ocean Dynamics*, 56:394–415, 2006.

[3] Hartmann T. and Wenzel H.-G. The HW95 tidal potential catalogue. *Geophysical Research Letters*, 22(24):3553–3556, 1995.

[4] Barthelmes F. and Förste C. The ICGEM-format. Technical report, GFZ Potsdam, Department 1 Geodesy and Remote Sensing, June 2011.

[5] Gegout P. http://gemini.gsfc.nasa.gov/aplo/Load_Love2_CM.dat, 1997.

[6] Munk W. H. and Cartwright D. E. Tidal Spectroscopy and Prediction. *Phil. Trans. Roy. Soc. London*, A(259):533–581, 1966.

[7] Marsh J. G., Lerch F. J., Putney B. H., Christodoulidis D. C., Felsentreger T. L., Sanchez B. V., Smith D. E., Klosko S. M., Martin T. V., Pavlis E. C., Robbins J. W., Williamson R. G., Colombo O. L., Chandler N. L., Rachlin K. E., G. B. Patel, Bhati S., and Chinn D. S. An Improved Model of the Earth's Gravitational Field: *GEM-T1*. Technical Report NASA Technical Memorandum 4019, Goddard Space Flight Center, Greenbelt, MD, 1987.

[8] Petit G. and Luzum B., editors. *IERS Conventions (2010)*. IERS Technical Note No. 36. Verlag des Bundesamts für Kartographie und Geodäsie, Frankfurt am Main, 2010.

[9] Dow J. M. Ocean Tides and Tectonic Plate Motions from Lageos. *Deutsche Geodätische Kommission*, Reihe C(344), 1988.

Appendix A: Harmonic synthesis of tidal heights

The reaction of the ocean on the Tide Generating Potential is the sum of the frequency-dependent tidal heigths ζ_s, which can be represented in terms of amplitude ξ_s and phase δ_s at a specific point on the Earth (λ, ϕ) at a distinct time t

$$\zeta(\lambda.\phi, t) = \sum_s \xi_s(\lambda, \phi) \cos\left[\Theta_s(t) + \chi_s - \delta_s(\lambda, \phi)\right]. \tag{1}$$

This formula is further depending on the astronomical (or Doodson) argument Θ_s and the Doodson-Warburg phase correction χ_s as defined in the IERS conventions 2010, Table 6.6 [8]. In a further step the tidal heights can be developed in terms of spherical harmonics [9]. For this, the tidal height of a particular tide in equation (1) is written as

$$\zeta_s = \xi_s \cos(\delta_s) \cos(\Theta_s + \chi_s) + \xi_s \sin(\delta_s) \sin(\Theta_s + \chi_s) \tag{2}$$

with the in-phase and quadrature expressions that can be developed in spherical harmonics expansions

$$\begin{aligned} \xi_s \cos(\delta_s) &= \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(a_{nm,s} \cos m\lambda + b_{nm,s} \sin m\lambda\right) P_{nm}(\sin\phi) \\ \xi_s \sin(\delta_s) &= \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(c_{nm,s} \cos m\lambda + d_{nm,s} \sin m\lambda\right) P_{nm}(\sin\phi) \end{aligned} . \tag{3}$$

Here, $a_{nm,s}, b_{nm,s}, c_{nm,s}, d_{nm,s}$ are spherical harmonics coefficients, while P_{nm} are Legendre polynomials in dependency of degree and order n, m. Inserting equation (3) in (2) and a re-arrangement applying trigonometric identities leads then to the representation with coefficients $C^{\pm}_{nm,s}, S^{\pm}_{nm,s}$

$$\zeta_s = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \sum_{+}^{-} \left[C^{\pm}_{nm,s} \cos(\Theta_s + \chi_s \pm m\lambda) + S^{\pm}_{nm,s} \sin(\Theta_s + \chi_s \pm m\lambda)\right] P_{nm}(\sin\phi) \tag{4}$$

or in terms of (tidal height) amplitudes $\hat{C}^{\pm}_{nm,s}$ and phases $\epsilon^{\pm}_{nm,s}$

$$\zeta_s = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \sum_{+}^{-} \hat{C}^{\pm}_{nm,s} \sin(\Theta_s + \chi_s \pm m\lambda + \epsilon^{\pm}_{nm,s}) P_{nm}(\sin\phi). \tag{5}$$

The coefficients, amplitudes and phases can be retrieved from each other by the following relations

$$\begin{aligned} C^{\pm}_{nm,s} &= \hat{C}^{\pm}_{nm,s} \sin(\epsilon^{\pm}_{nm,s}) = \frac{1}{2}(a_{nm,s} \mp d_{nm,s}) \\ S^{\pm}_{nm,s} &= \hat{C}^{\pm}_{nm,s} \cos(\epsilon^{\pm}_{nm,s}) = \frac{1}{2}(c_{nm,s} \pm b_{nm,s}) \end{aligned} . \tag{6}$$

The mass redistribution effect of ocean tides on the Earth's gravitational potential is described by

$$\Delta V^O_s = \frac{GM}{R} \sum_{n=0}^{\infty} \left(\frac{R}{r}\right)^{n+1} \sum_{m=0}^{n} \left[\Delta c_{nm,s} \cos(m\lambda) + \Delta s_{nm,s} \sin(m\lambda)\right] P_{nm}(\sin\phi). \tag{7}$$

G is denoting the gravitational constant, M the mass of the Earth, R the Earth's radius and $\Delta c_{nm,s}$, $\Delta s_{nm,s}$ the dimensionless potential coefficients that can be retrieved by

$$\begin{aligned} \Delta c_{nm,s} &= \frac{4\pi R^2 \rho_w}{M} \frac{1 + k'_n}{2n+1} \left[(C^{+}_{nm,s} + C^{-}_{nm,s}) \cos(\Theta_s + \chi_s) + (S^{+}_{nm,s} + S^{-}_{nm,s}) \sin(\Theta_s + \chi_s)\right] \\ \Delta s_{nm,s} &= \frac{4\pi R^2 \rho_w}{M} \frac{1 + k'_n}{2n+1} \left[(S^{+}_{nm,s} - S^{-}_{nm,s}) \cos(\Theta_s + \chi_s) - (C^{+}_{nm,s} - C^{-}_{nm,s}) \sin(\Theta_s + \chi_s)\right] \end{aligned} \tag{8}$$

with the water density ρ_w and the load love numbers k'_n. Introducing the factor $1 + k'_n$ therefore accounts for both, the mass effect of ocean tides, as well as the effect of the deformation of the solid Earth as a consequence of ocean mass redistribution.