Gravitational effect of ocean and atmospheric tides

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The gravitational effect of ocean and atmospheric tides including loading effects at a given time can be represented as corrections to the Stokes coefficients of a spherical harmonics expansion. This requires not only an atlas of selected major tidal consitutents but also a calculation rule to complement all frequency lines. This can be done by interpolation using an admittance matrix. For this purpose, a file format and an algorithm are defined here.

Data Format

An ocean/atmospheric tide model consists of the following files:

- `<model>_001fileList.txt`: Two file names per row for each tidal line k of the tide atlas, first name for the cos-component, second name for the sin-component.
- `<model>_002doodson.txt`: A matrix $\mathbf{D} = (D_{f,i})$ with $i = 1 \dots 6$ integer multipliers per row f.
- `<model>_003admittance.txt`: A matrix $\mathbf{A} = (A_{k,f})$ with one line for each row k. The number of columns corresponds to the number of rows of the Doodson matrix before.
- `<model>_<doodson>_<name>_<cos/sin>.gfc`: One file for each cos and sin component for each tidal line k as given in the file list at top. The files uses the ICGEM-format for Earth Gravity Field Models (`product_type gravity_field`). Please note: The ICGEM defines also a format for ocean tides, which is not used here.

All files are free formated: The elements of each lines do not have a fixed length and are separated by white spaces.

Synthesis

The dynamical effects of ocean and atmpospheric tides including loading effects are most easily incorporated as periodic variations in the normalized Stokes' coefficients $\Delta\bar{C}_{nm}(t)$ and $\Delta\bar{S}_{nm}(t)$ of degree n and order m. The following steps have to be performed:

1. The phase argument $\theta_f(t)$ of all considered major, minor and nonlinear tides f can be computed with

$$\theta_f(t) = \sum_{i=1}^{6} D_{f,i}\,\beta_i(t), \tag{1}$$

where $\mathbf{D} = (D_{f,i})$ is a matrix with 6 columns containing the multipliers in a row for each tidal frequency f provided in the file `<model>_002doodson.txt`. The six-vector $\beta(t) = (\tau, s, h, p, N_0, p_s)$ contains the Doodson's fundamental arguments computed at time t.

2. The modulated periodic factors for the cos/sin components of the model tides are computed using an admittance matrix $\mathbf{A} = (A_{k,f})$ in the file `<model>_003admittance.txt`:

$$f_{k,cos}(t) = \sum_f A_{k,f} \cos\theta_f(t) \quad \text{and} \quad f_{k,sin}(t) = \sum_f A_{k,f} \sin\theta_f(t). \tag{2}$$

3. The periodic variations in the normalized Stokes' coefficients can be evaluated as

$$\begin{bmatrix}\Delta\bar{C}_{nm}(t)\\ \Delta\bar{S}_{nm}(t)\end{bmatrix} = \sum_k f_{k,cos}(t) \begin{bmatrix}\bar{C}_{nm}\\ \bar{S}_{nm}\end{bmatrix}_{k,cos} + f_{k,sin}(t) \begin{bmatrix}\bar{C}_{nm}\\ \bar{S}_{nm}\end{bmatrix}_{k,sin}. \tag{3}$$

The $\left[\bar{C}_{nm}, \bar{S}_{nm}\right]_k$ are normalized Stokes' coefficients of the model for each tidal line provided separately for the cos and sin component provided in the files `<model>_<doodson>_<name>_<cos/sin>.gfc`.

4. The tide models are usually given in the Center of Figure (CF) frame. For orbit computations in the Center of Mass (CM) frame the degree 1 coefficients $\Delta\bar{C}_{10}, \Delta\bar{C}_{11}, \Delta\bar{S}_{11}$ must be set to zero.

Atmospheric tides

Atmospheric tides can be handled in the same way as ocean tides and the synthesized Stokes' coefficients at time t must be added. The admittance matrix in this case is simple the identity matrix and can be omitted. It is only provided for convenience.

Software

For the convenience of the users, reference implementations in Python, MATLAB, and Fortran are supplied to the public domain (unlicense).